On three domination-based identification problems on block graphs*

Dipayan Chakraborty[†]

— joint work with

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Basic notations & definitions

$G = (V(G), E(G))$ is a graph and $v \in V(G)$.

N(v): the set of all neighbors of v in G.

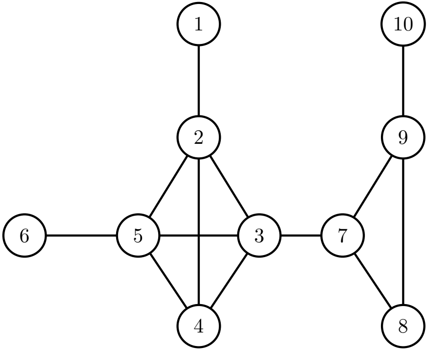
$\mathbf{N[v]} = N(v) \cup \{v\}$

$D \subset V(G)$ of a graph G is called a **Dominating set** of G if $\forall v \in V(G),\ N[v] \cap D \neq \emptyset$.

Identifying code (ID-Code) [Karpovsky et. al., 1998]

Locating-dominating code (LD-Code) [Slater, 1987]

Open-locating-dominating code (OLD-Code) [Seo & Slater, 2010]



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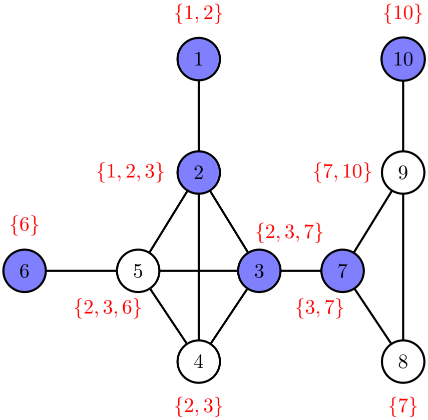

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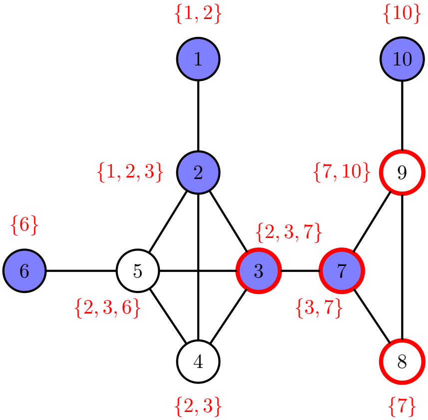

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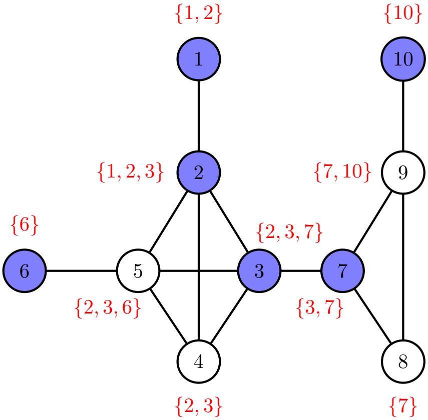

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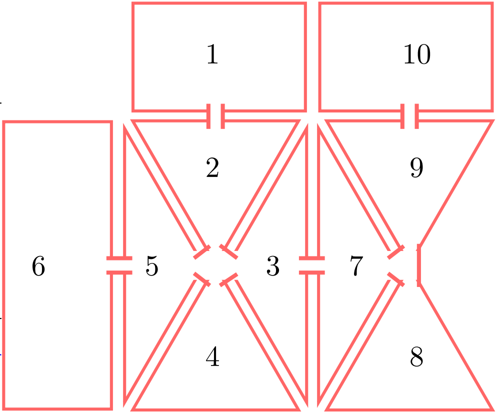

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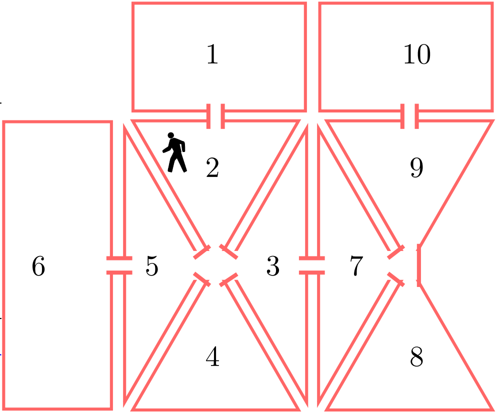

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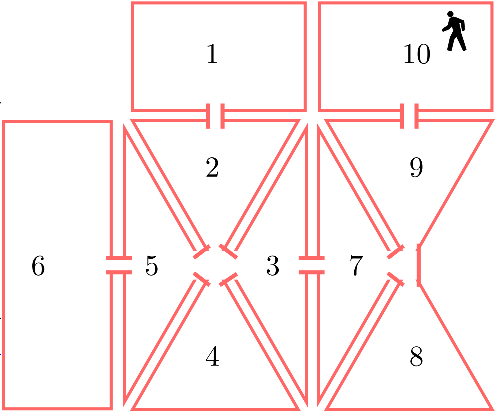

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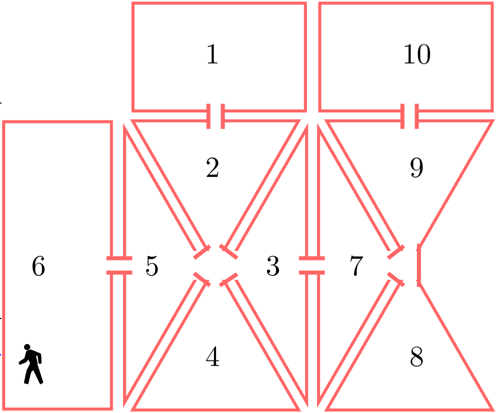

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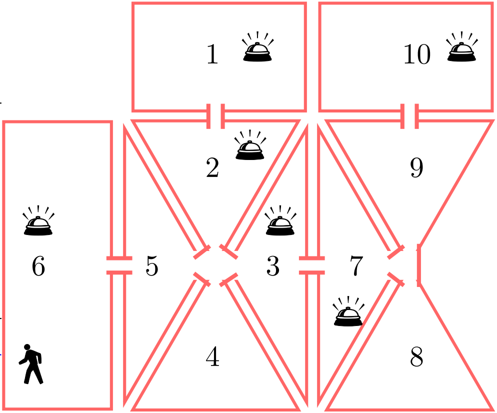

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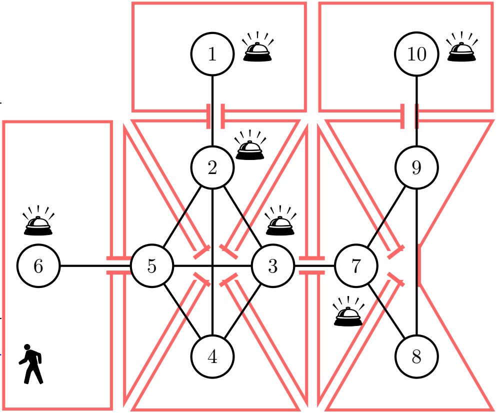

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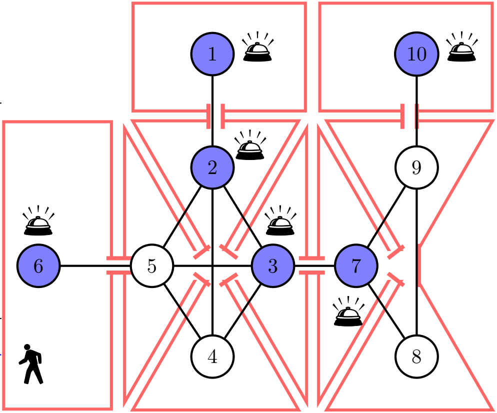

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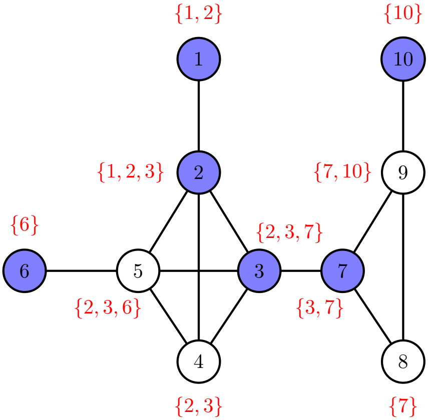

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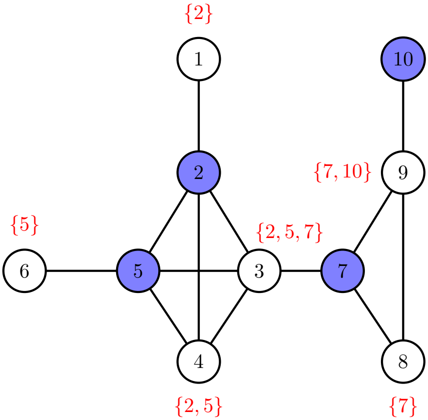

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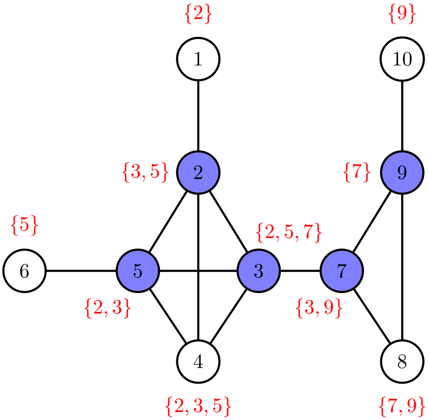


Identifying code (ID-Code) [Karpovsky et. al., 1998]

- ID-number $\gamma^{ID}(G) = \min |B|$ $\forall$ ID-Code B of G

Locating-dominating code (LD-Code) [Slater, 1987]

- LD-number $\gamma^{LD}(G) = \min |B|$ $\forall$ LD-Code B of G

Open-locating-dominating code (OLD-Code) [Seo & Slater, 2010]

- OLD-number $\gamma^{OLD}(G) = \min |B|$ $\forall$ OLD-Code B of G

Identifying code (ID-Code) [Karpovsky et. al., 1998]

- ID-number $\gamma^{ID}(G) = \min |B|$ $\forall$ ID-Code B of G

Exists only if G is closed twin-free.

Locating-dominating code (LD-Code) [Slater, 1987]

- LD-number $\gamma^{LD}(G) = \min |B|$ $\forall$ LD-Code B of G

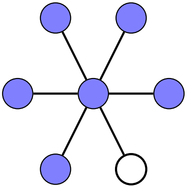
Always exists!

Open-locating-dominating code (OLD-Code) [Seo & Slater, 2010]

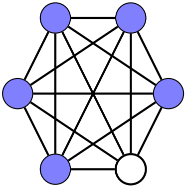
- OLD-number $\gamma^{OLD}(G) = \min$ $|B|$ $\forall$ OLD-Code B of G

Exists only if G is open twin-free and has no isolated vertices.

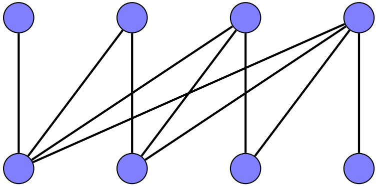
Some examples of code numbers



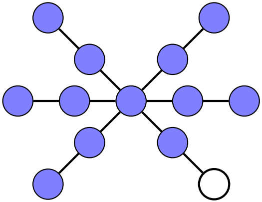
(a) $\gamma^{ID}(St_6) = \gamma^{LD}(St_6) = 5$



(b) $\gamma^{LD}(K_6) = \gamma^{OLD}(K_6) = 5$

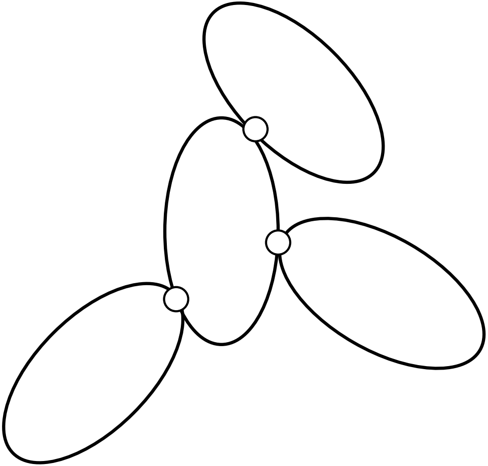


(a) $\gamma^{OLD}(HG) = 8$



(b) $\gamma^{OLD}(SSt_6) = 12$

Block graph



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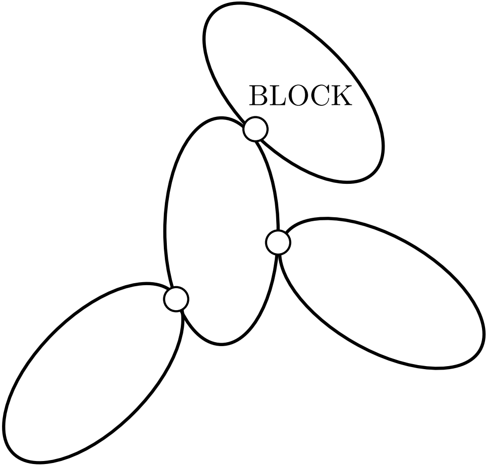

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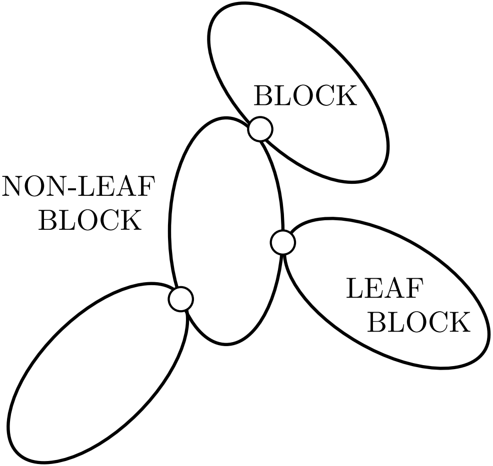

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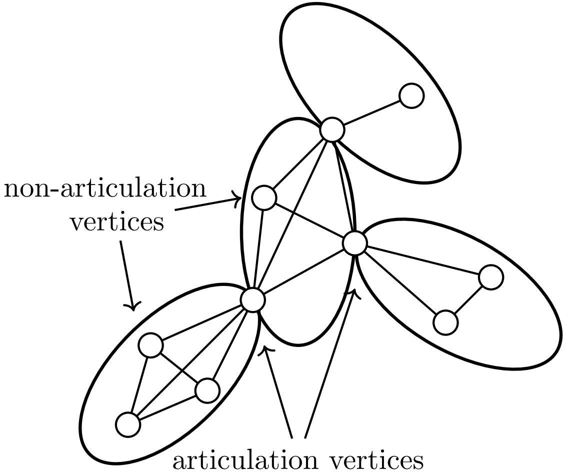
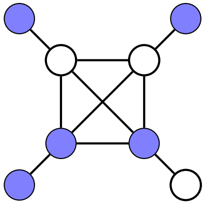


Results

Theorem (Conjecture. Argiroffo et. al. (2018))

Let G be a closed twin-free block graph. Then $\gamma^{ID}(G) \leq n_Q(G)$, where $n_Q(G)$ is the number of blocks of G.



Results

Theorem

Let G be a twin-free block graph without isolated vertices. Then, $\gamma^{LD}(G) \leq \frac{1}{2}|V(G)|$.

Conjecture. Garijo et. al (2014): Let G be a twin-free graph without isolated vertices. Then, $\gamma^{LD}(G) \leq \frac{1}{2}|V(G)|$.

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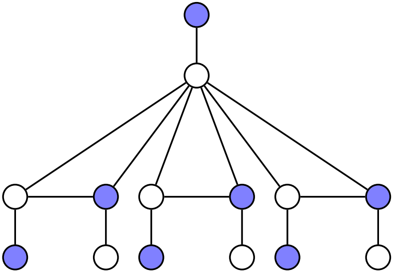

Results

Theorem

Let G be a connected open twin-free block graph, with no isolated vertices and $G \ncong P_2,\ P_4$. Let $m_Q(G)$ be the number of non-leaf blocks with at least one non-articulation vertex. Then, $\gamma^{OLD}(G) \leq |V(G)| - 1 - m_Q(G)$.

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In general: For an open twin-free graph G, $\gamma^{OLD}(G) \leq |V(G)| - 1$.
Foucaud et. al. (2021): Unless G is a half-graph, in which case, $\gamma^{OLD}(G) = |V(G)|$.

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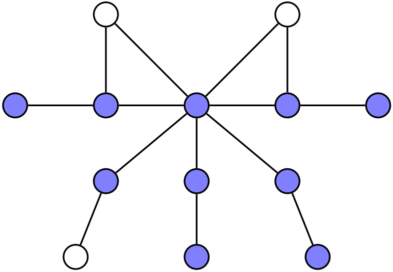

Results

Theorem

Let G be a connected block graph. Then

- $\gamma^{ID}(G) \geq \frac{|V(G)|}{3} + 1$,
- $\gamma^{LD}(G) \geq \frac{|V(G)|+1}{3}$, and
- $\gamma^{OLD}(G) \geq \frac{|V(G)|}{3} + 1$ (except when $G \cong K_4$ with 3 leaves on distinct support vertices).

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General lower bound: $\gamma^{ID}(G)$, $\gamma^{LD}(G)$, $\gamma^{OLD}(G) \geq \lceil \log_2(|V(G)| + 1) \rceil$.

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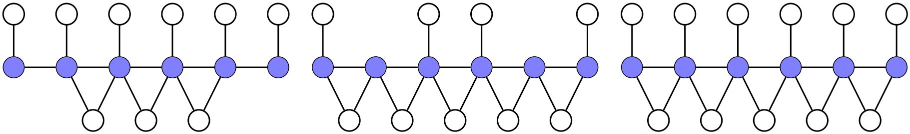

(a) $\gamma^{ID}(G) = 6$, $|V(G)| = 15$ **(b)** $\gamma^{OLD}(G) = 6$, $|V(G)| = 15$ **(c)** $\gamma^{LD}(G) = 6$, $|V(G)| = 17$

Results

Theorem

Let G be a connected block graph. Then

- $\gamma^{ID}(G) \geq \frac{3(n_Q(G)+2)}{7}$,
- $\gamma^{LD}(G) \geq \frac{n_Q(G)+2}{3}$, and
- $\gamma^{OLD}(G) \geq \frac{n_Q(G)+3}{2}$.

Results

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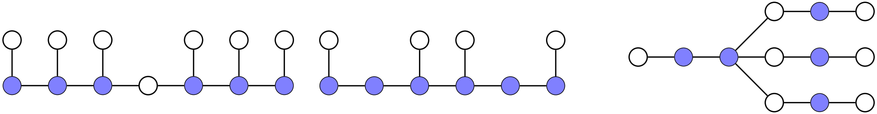

(a) $\gamma^{ID}(G) = 6$, $|V(G)| = 13$ **(b)** $\gamma^{OLD}(G) = 6$, $|V(G)| = 10$ **(c)** $\gamma^{LD}(G) = 5$, $|V(G)| = 12$

Thank you!